STRONGWELL GRIDFORM SLAB DESIGN MANUAL

LAWRENCE C. BANK

MICHAEL G. OLIVA

JEFFREY J. BRUNTON

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REFERENCES
1 INTRODUCTION

The Strongwell Gridform Slab Design Manual is provided to supplement and clarify the Strongwell Gridform Slab Design Program for the design of reinforced concrete bridge deck slabs reinforced with double layer 3D pultruded Gridform. The program was developed using the following codes: AASHTO LRFD (2007), ACI 440.1R-06, and a draft of the AASHTO LRFD Bridge Design Guide Specifications for GFRP Reinforced Concrete Decks and Traffic Railing (AASHTO FRP). General bridge design procedures of the AASHTO LRFD Bridge Design Specifications are included in the program and the design checks were completed using ACI 440 and the AASHTO FRP Draft. Those procedures include loading, load combinations, and allowable service deflections. The design checks in this program are the same for ACI 440 and the AASHTO FRP Draft except the following items noted in this section. The AASHTO FRP Draft limits the concrete stress under the Service I limit state to 45% of the concrete strength. This provision was included in the design program. ACI 440 does not include this limit state. Fatigue and creep rupture is required to be checked in the ACI 440 code. The AASHTO FRP Draft requires that fatigue and creep rupture be investigated for slabs unless the slab is in a "multi-girder" application. The stress limit for fatigue and creep rupture is the same. The option of checking for fatigue and creep rupture stress is available regardless of the slab support configuration.

An example is provided with the equations used in the program. The program has a few limitations and general items that are discussed on the input sheet within the program. There are parametric tables that can be viewed for optimum spacing of main reinforcement bar sizes. The program contains the AASHTO FRP / ACI 440 punching shear equation and the University of Wisconsin-Madison punching shear equation. The recommended punching shear equation used for design is the University of Wisconsin-Madison equation specifically developed for the Gridform system.
2 GENERAL DESIGN NOTES

1. Limit the maximum aggregate size in concrete. A Wisconsin Grade D Size 1 (3/4 inch maximum aggregate) was used on all test slabs. A ¾ inch maximum aggregate size is recommended for all deck slabs to make sure the concrete properly covers all reinforcement.

2. Place primary reinforcement perpendicular to supporting girders and traffic.

3. The program does not consider cantilevered slabs. See AASHTO LRFD (2007) Section 3.6.1.3.4 for details.

4. Top reinforcement must be in alignment with bottom reinforcement.

5. Beam shear is not a design criterion because current AASHTO FRP and ACI 440 design equations underestimate the beam shear capacity of concrete slabs reinforced with Gridform.

3 INPUT SHEET

The input sheet is the user’s interface for the design program. Options and user input parameters are limited to this sheet. Read the “Program Design Basics” and “Instructions for use of Excel Sheets” in the input sheet prior using the program. All the necessary data required to be placed in the program is located on this sheet. A design analysis will be completed for the bridge deck design code compliance from the options and parameters selected on this sheet.

3.1 Design Basics

3.1.1 Type of Bridge

(AASHTO LRFD Section 4.6.2.1.6)

This section considers the type of girders that will be used in the bridge. The design negative moment is related to distance from the centerline of the girder to the critical section, and the critical section is based on the type of girder. The negative moment decreases as the critical
negative moment section moves farther away from the centerline of the supporting girder. The critical negative moment offset is indicated below for each bridge type.

1. Monolithic construction and concrete box beams - at the face of the supporting component
2. Steel and wood beams - at one-quarter of the flange width from the centerline of the support
3. Precast I-shaped and T-shaped concrete beams - at one third of flange width from the centerline, but not exceeding 15 inches from the centerline of support

Design Example: Type of Bridge Specification
Steel beams - at one-quarter of the flange width from the centerline of the support

3.1.2 Environmental Reduction Factor

(ACI 440 Table 7.1 and AASHTO FRP Table 2.6-1)

The guaranteed material properties do not include the effect of long term environmental exposure. The environmental reduction factor accounts for the uncertainties and reduction of material properties due to environmental exposure over the life of the FRP reinforced concrete members. The ultimate tensile strength and design tensile strain is be reduced by the environmental factor. Select the appropriate exposure condition. The environmental reduction factor is also dependent on the type of fiber. The Gridform system is glass FRP. For each exposure condition the environmental reduction factor is presented below:

1. Concrete exposed to earth and weather – \( C_E = 0.7 \)
2. Concrete not exposed to earth and weather – \( C_E = 0.8 \)

Design Example: Environmental Reduction Factor Specification
Concrete exposed to earth and weather – \( C_E = 0.7 \)
3.1.3 Fatigue and Creep Rupture Limit State

(AASHTO FRP Section 2.7.3)

The AASHTO FRP draft limits the stress in the FRP tensile reinforcement for non-multigirder applications. Statically determinant slabs must be checked for fatigue and creep rupture for the Service I loading condition (AASHTO LFRD, 2007). ACI 440 requires the fatigue and creep rupture limit state for the Service I moments.

Design Example: Fatigue and Creep Rupture Limit State Specification

Fatigue and creep rupture check is necessary

3.2 Properties

3.2.1 FRP Material Properties

This section suggests the guaranteed material properties for use in the design program. ACI 440 Section 3.2 and AASHTO FRP Section 4.9 were used as guidance for determining material properties. ACI 440 requires a minimum of 25 samples to determine the properties. AASHTO FRP requires a minimum of 5 samples to be tested from 5 production lots. Thirty main bar samples and five cross rod samples were tested by Strongwell to determine the properties. The suggested guaranteed design values (SGDV) for tensile strength and tensile strain are the average minus three standard deviations from test data. The SGDV modulus of elasticity is the average from the test samples. Table 1 presents the suggested guaranteed design values. The material property tests results for the main bars were provided by Strongwell, and the material property test results for the cross rods were obtained from Conachen (2005). The main bars and cross rods have different material properties.
Table 1 - Suggested Guaranteed Design FRP Material Properties

<table>
<thead>
<tr>
<th>FRP Section</th>
<th>Tensile Strength (ksi)</th>
<th>Tensile Modulus (ksi)</th>
<th>Ultimate Tensile Strain (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Bars</td>
<td>Average: 100.8</td>
<td>5191</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>St. Dev.: 10.0</td>
<td>262</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>SGDV: 70.8</td>
<td>5191</td>
<td>0.014</td>
</tr>
<tr>
<td>Cross Rods</td>
<td>Average: 160.5</td>
<td>6928</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>St. Dev.: 2.5</td>
<td>88</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>SGDV: 153</td>
<td>6930</td>
<td>n/a</td>
</tr>
</tbody>
</table>

\[ f_{u,bar} = \text{main reinforcement ultimate tensile strength} \]

\[ f_{u,cr} = \text{cross rod ultimate tensile strength} \]

\[ \varepsilon_{u,bar} = \text{main reinforcement ultimate tensile strain} \]

\[ E_{bar} = \text{modulus of elasticity for main reinforcement} \]

\[ E_{cr} = \text{modulus of elasticity for cross rods} \]

Design Example: FRP Material Properties Selection

Testing on the material properties was completed by Strongwell, and Quality Assurance testing was completed by Conachen (2005). Data for the design example came from both of these sources. The main bar properties are the average material properties for the testing completed by Conachen (2005), and the cross rod material properties are the average material properties for testing completed by Strongwell. Do not use these values for actual design.

\[ f_{u,bar} = 104 \text{ ksi} \]

\[ f_{u,cr} = 160.4 \text{ ksi} \]

\[ \varepsilon_{u,bar} = 0.01 \]

\[ E_{bar} = 4750 \text{ ksi} \]

\[ E_{cr} = 6920 \text{ ksi} \]
3.2.2 Slab Input Properties

This section requires the user to place in the basic slab properties. Figure 1 shows the dimensions presented in this section. The slab input properties are presented below:

\[ h = \text{slab thickness} \]

The minimum slab thickness is 7 inches (AASHTO LRFD Section 9.7.2).

\[ s(\text{slab span}) = \text{distance from centerline to centerline of supporting girders} \]

The design program uses the HL-93 live load design moment from Table A4-1 (AASHTO LRFD, 2007). The spans have to be within the span range presented in the table that is between 4 feet and 15 feet to determine the live load moment. The maximum laboratory test distance from centerline to centerline of the supporting elements (girders) was 8.5 feet, which is the maximum recommended girder spacing for Gridform. Spans beyond 8.5 feet have not been investigated for failure modes.

\[ \gamma_c = \text{unit weight of concrete} \]

The spreadsheet was developed for normal weight concrete. The self-weight and modulus of elasticity are dependent on the unit weight of concrete.

\[ w_{fl} = \text{width of supporting girder top flange} \]

\[ c_{\text{top}} = \text{top clear cover of reinforcement} \]

\[ c_{\text{bottom}} = \text{bottom clear cover of reinforcement} \]

The recommended minimum top clear cover of reinforcement is 1.5 inches. The recommended minimum bottom clear cover is 1 inch for Gridform without stay in place formwork and 0 inches with Gridform with stay in place formwork. The recommendations for the clear cover were based off the same clear covers used in the laboratory testing of slabs reinforced with Gridform.
clear = clear distance between top and bottom main reinforcement bars

\[ = h - (c_{\text{top}} + c_{\text{bottom}} + 2 \times \text{bar height}) \geq 1.5 \text{ in} \]

The minimum recommended clear distance between the top and bottom layers of main reinforcement bars is the same as the test slabs of 1.5 inches. The ACI 440 and AASHTO FRP do not cover the minimum spacing between the bars.

**Figure 1 – Slab Dimensions**

<table>
<thead>
<tr>
<th>Design Example: Slab Input Properties Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 8 \text{ in} )</td>
</tr>
<tr>
<td>( s = 93.7 \text{ in} )</td>
</tr>
<tr>
<td>( \gamma_c = 150 \text{ pcf} )</td>
</tr>
<tr>
<td>( w_{fi} = 20 \text{ in} )</td>
</tr>
</tbody>
</table>
\[ c_{top} = 1.5 \text{ in} \]
\[ c_{bottom} = 1.0 \text{ in} \]
\[ \text{clear} = 8\text{ in} - (1.5\text{ in} + 1\text{ in} + 2 \times 1.5\text{ in}) = 2.5 \text{ in} \geq 1.5 \text{ in} \quad (1.5” \text{ I bars}) \]

### 3.3 Design Loads

#### 3.3.1 Point Load Specification

The point load for punching shear and deflection calculations is indicated in this section. The default point load \((P_{LL})\) 16 kips, which is half of the axle load as indicated in Section 3.6.1.2.2 (AASHTO LRFD, 2007). The dynamic load allowance factor \((1+IM)\) default is 1.33 that is stipulated by Table 3.6.2.1-1 (AASHTO LRFD, 2007).

#### 3.3.2 Tire Contact Area Specification

The tire contact dimensions are specified in Section 3.6.1.2.5 (AASHTO LRFD). Designer may specify other contact dimensions, but the dimensions must be within code specifications. The tire contact area is used to calculate the critical punching shear perimeter. The AASHTO LRFD specified dimensions of the width \((w)\) and length \((l)\) of tire contact area are 20 inches and 10 inches respectively.

#### 3.3.3 Uniform Load Specification and Input

This section specifies the uniform loading on the slab. The following terms are presented below:

\[ DL_{sw}(\text{concrete deck weight}) = h \times \gamma_c \]

The self-weight of the concrete deck is automatically calculated.

\[ DL_{si} = \text{superimposed dead load} \]

Specify the weight of a future wearing surface or any other uniform dead load.

\[ LL_{ui} = \text{uniform live load} \]
Select a uniform live load to be placed on the deck. The uniform live load is superimposed on top of the HL-93 loading, which will be in addition to loading from the load table.

\[ LL_{\text{lane}} = \text{lane loading for the design truck} \]

Enter the uniform lane loading. AASHTO LRFD requires a uniform lane load of 640 plf distributed over 10 feet (64 psf). The tables that select the appropriate design moments already consider the uniform lane load. The uniform lane loading is only used for deflection purposes where the maximum live load deflection is determined.

\[ \text{Coeff}_{\text{pos}} = \text{moment coefficient in equation} \left( c \times (wL^2) \right) \]

\[ \text{Coeff}_{\text{neg}} = \text{moment coefficient in equation} \left( c \times (wL^2) \right) \]

See specific state department of transportation for positive and negative moment coefficients. The coefficients will be applied to uniform loads. Exterior and interior spans will have the same main reinforcement bars. It is only necessary to consider the exterior spans or the maximum positive and negative moment coefficient.

**Design Example: Design Load Specification**

\[ P_{LL} = 16 \text{ kip} \]

\[ 1 + IM = 1.33 \]

\[ w = 20 \text{ in} \]

\[ l = 10 \text{ in} \]

\[ DL_{sw} = 8 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \times 150 \text{pcf} = 100 \text{ psf} \]

\[ DL_{si} = 0 \text{ psf} \]

\[ LL_{u} = 20 \text{ psf} \]
Uniform live load was only included for demonstration purposes only. Reference appropriate code to determine if additional live loads are required.

\[ LL_{lane} = 64 \text{ psf} \]

\[ \text{Coeff}_{pos} = 0.080 \]

\[ \text{Coeff}_{neg} = 0.100 \]

### 3.3.4 Load Combinations

The required load combinations are programmed into the spreadsheet. If other load combinations are desired, insert load factors for the dead and live loads. The load factors for the “other” load combinations will be used for the strength limit states only (moment capacity and punching shear). The Strength I (flexure and punching shear) and Service I (fatigue and creep rupture, cracking, concrete stress, and deflection) load combinations are the required by AASHTO LRFD (Table 3.4.1-1). The load factors are presented in Table 2. Leave the “other” load combination blank if the “other” load combination is not desired.

#### Table 2 – Load Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>LL</th>
<th>DL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I</td>
<td>1.75</td>
<td>1.25</td>
</tr>
<tr>
<td>Service I</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Other</td>
<td>Designer’s choice</td>
<td>Designer’s choice</td>
</tr>
</tbody>
</table>

### 3.4 Crack Width Calculation

Specify the maximum allowable crack width (\(w_{allow}\)). The maximum allowable crack width (AASHTO FRP Section 2.9.3.4 and ACI 440 Section 8.3.1) is 0.02 inches. Section 8.3.1 also states “The provisions specified herein shall apply to the reinforcement of all concrete components, except those structures for which aesthetic is not a concern.” If aesthetics are not a concern, select the option in the dropdown menu and keep the \(w_{allow}\) blank. See AASHTO FRP Section 2.9.3.4 and ACI 440 Section 8.3.1 for more information regarding the bond coefficient
(k_b). Testing indicates that the range of all bond coefficients is 0.60 to 1.72. A value 1.0 means that FRP and steel have the same bond. Values greater than 1.0 mean that FRP has a weaker bond than steel in conjunction with concrete. ACI 440 states that the average is 1.1 and a conservative value for design is 1.4. The recommended bond coefficient for Gridform is 1.4.

Design Example: Crack Width Criteria

Aesthetics is a concern

\[ W_{allow} = 0.02 \text{ in} \]

\[ k_b = 1.4 \]

### 3.5 Deflection Limits

The deflection of bridge deck slabs is controlled by Section 9.5.2 (AASHTO LFRD 2007). The acceptable live load deflections are presented below and are dependent on the pedestrian traffic on the bridge.

1. \( L/800 \) - No pedestrian traffic
2. \( L/1000 \) - Limited pedestrian traffic
3. \( L/1200 \) – Significant pedestrian traffic

Design Example: Deflection Requirements

L/800 – No pedestrian traffic

\[ 1/x=1/800=0.00125 \]

\[ \Delta_{limit-LL} = \frac{1}{800} \times 93.7 \text{ in} = 0.117 \text{ in} \]

### 3.6 Design

Select the concrete strength, bar spacing and main bar size. Check the clear distance between the bars. The maximum recommended clear distance is 1.5 inches as previously mentioned in Section 3.2.2.
Design Example: concrete strength, bar spacing and main bar size selection

\[ f'_c = 5000 \text{ psi} \]

\[ b_s = 4 \text{ in} \]

1.5” I bar

3.7 Results

The design code check compliance results are presented for the following limit states:

1. Temperature and shrinkage – AASHTO FRP and ACI 440 have the identical requirements.
2. Positive and Negative Moment - AASHTO FRP and ACI 440 have the identical requirements.
3. Punching Shear - AASHTO FRP and ACI 440 have the identical requirements. The punching shear strength estimations are conservative. The University of Wisconsin punching shear equation is the suggested punching shear design equation.
4. Fatigue and Creep Rupture - AASHTO FRP and ACI 440 have the same predictions for the stress in the reinforcement and limit stress in the reinforcement. ACI 440 requires that the fatigue and creep rupture be investigated for all slabs. AASHTO FRP does not require that fatigue and creep rupture analysis for slabs that are in “multi-girder” applications.

4 CALC SHEET

The calculation sheet contains preliminary calculations. The contents of the input sheet are located in this sheet. Only new items will be discussed in the design manual. The following terms from the FRP properties section are listed below:

\[ C_E = \text{environmental reduction factor} \]
The equations for the design tensile strengths and strains are located in AASHTO FRP Section 2.6.1.2 and ACI 440 Section 7.2.

\[ f_{d \text{ bar}} = \text{main reinforcement design tensile strength} = C_E \times f_{u \text{ bar}} \]

\[ f_{d \text{ cr}} = \text{cross rod design tensile strength} = C_E \times f_{u \text{ cr}} \]

\[ \varepsilon_{d \text{ bar}} = \text{main reinforcement design tensile strain} = C_E \times \varepsilon_{u \text{ bar}} \]

\[ \varepsilon_c = \text{ultimate concrete strain} = 0.003 \]

**Design Example: Material Properties**

\[ C_E = 0.7 \]

\[ f_{d \text{ bar}} = 0.7 \times 104000 \, \text{psi} = 72800 \, \text{psi} \]

\[ f_{d \text{ cr}} = 0.7 \times 160400 \, \text{psi} = 112280 \, \text{psi} \]

\[ \varepsilon_{d \text{ bar}} = 0.7 \times 0.01 = 0.007 \]

### 4.1 Unfactored HL-93 Moments

The Unfactored HL-93 moments looks up the corresponding HL-93 live load moments from Table A4-1. The row link for positive moment is rounded up to the nearest inch because the table has the centerline to centerline of the span in 1 inch increments. The row link for the negative moment link is a cell linked to the type of bridge. \( F_{\text{offset}} \) is the critical negative moment distance from the centerline of the girder calculated using the negative moment link cell.

**Design Example: HL-93 Moments**

\[ \text{row link} = \, s = \text{round up} (93.7 \, \text{in}) = 94 \, \text{inches} \]

\[ F_{\text{offset}} = \text{round down} (0.25 \times w_f) = 0.25 \times 20 \, \text{in} = 5 \, \text{in} \]

From AASHTO LRFD Table A4-1
4.2 Moment Calculations

The moment calculations section determines the live and dead load design moments. The HL-93 and uniform live load moment are added together. The dead load moments are determined by using the moment coefficients and the dead load multiplied by the square of the span length.

**Design Example: Moment Calculations**

The live load moments are the HL-93 live load moments because there is no uniform live load. The self-weight contributes to the dead load. The design example design moment calculations are located in Table 3.

\[
M_{pos \text{HL–93}} = 5.60 \text{kip} \frac{ft}{ft}
\]

\[
M_{neg \text{HL–93}} = -5.02 \text{kip} \frac{ft}{ft}
\]

\[
M_{pos LLu} = 0.080 \times .020 \text{ksf} \times (93.7 \text{ in})^2 \times \frac{1\text{ft}^2}{144\text{in}^2} = 0.10 \text{kip} \frac{ft}{ft}
\]

\[
M_{neg LLu} = -0.1 \times .020 \text{ksf} \times (93.7 \text{ in})^2 \times \frac{1\text{ft}^2}{144\text{in}^2} = -0.12 \text{kip} \frac{ft}{ft}
\]

\[
M_{pos \text{total}} = 5.60 \text{kip} \frac{ft}{ft} + 0.10 \text{kip} \frac{ft}{ft} = 5.70 \text{kip} \frac{ft}{ft}
\]

\[
M_{neg \text{total}} = -5.02 \text{kip} \frac{ft}{ft} - 0.12 \text{kip} \frac{ft}{ft} = -5.14 \text{kip} \frac{ft}{ft}
\]
\[ M_{\text{pos \, DL \, total}} = M_{\text{pos \, DL \, self}} = 0.080 \times 0.100 \, \text{ksf} \times (93.7 \text{ in})^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 0.49 \, \text{kip ft} \]

\[ M_{\text{neg \, DL \, total}} = M_{\text{neg \, DL \, self}} = -0.1 \, 0 \times 0.100 \, \text{ksf} \times (93.7 \, \text{in})^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = -0.61 \, \text{kip ft} \]

**Table 3 – Design Example Moment Calculations**

<table>
<thead>
<tr>
<th>Moment Calculations (kip-ft/ft)</th>
<th>Live</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M\text{pos} (Strength I)</td>
<td>5.60</td>
<td>-5.02</td>
<td></td>
</tr>
<tr>
<td>M\text{neg} (Strength I)</td>
<td>0.10</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.70</td>
<td>-5.14</td>
<td></td>
</tr>
</tbody>
</table>

**4.3 Moments and Ultimate Design Moments**

These sections calculate the moments for the design load combinations. The moments for Strength I, Service I, and the other load combinations are determined.

**Design Example: Moments and Ultimate Design Moments**

The design example design moments are located in Table 4.

**Table 4 – Design Example Moments**

<table>
<thead>
<tr>
<th>Moment</th>
<th>Load Factor</th>
<th>M\text{LL} (kip-ft/ft)</th>
<th>Load Factor</th>
<th>M\text{DL} (kip-ft/ft)</th>
<th>M\text{U} (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M\text{pos} (Strength I)</td>
<td>1.75</td>
<td>9.98</td>
<td>1.25</td>
<td>0.61</td>
<td>10.59</td>
</tr>
<tr>
<td>M\text{neg} (Strength I)</td>
<td>1.75</td>
<td>-9.00</td>
<td>1.25</td>
<td>-0.76</td>
<td>-9.76</td>
</tr>
<tr>
<td>M\text{pos} (Service I)</td>
<td>1.00</td>
<td>5.70</td>
<td>1.00</td>
<td>0.49</td>
<td>6.19</td>
</tr>
<tr>
<td>M\text{neg} (Service I)</td>
<td>1.00</td>
<td>-5.14</td>
<td>1.00</td>
<td>-0.61</td>
<td>-5.75</td>
</tr>
<tr>
<td>M\text{pos} (Other)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>M\text{neg} (Other)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
4.4 Parametric Bar Spacing

The design program will determine the maximum main bar spacing from the parametric bar spacings for each code requirement. The maximum main bar spacing will be calculated for all main member sizes and the following concrete strengths: 4000 psi, 5000 psi, 6000 psi, 7000 psi, and 8000 psi. The following bar spacings are the suggested main bar spacings from Strongwell: 3, 4, 4.5, 5, 6, and 7 inch spacings. Other main bar spacing are possible; contact Strongwell for other bar spacings.

5 Design Results Sheet, Parametric Design Sheet, and Parametric Results Sheet

These sheets contain the design code checks. The design results sheet contains the results for the selected options in the design selection of the input sheet. The parametric design sheet determines the maximum bar spacing allowed for the parametric bar spacings on the calc sheet for each main bar and concrete strength. The parametric results summarized in the parametric results sheet.

5.1 Material Properties

This section includes only the terms that have not been introduced in the design manual to this point.

\[ f'_{c} = \text{concrete compressive strength} \]

\[ A_g = \text{gross area of main reinforcement} \]

The design material properties are transferred to this section.

\[ E_c = \text{modulus of elasticity for concrete} = 33000 \times \gamma_c^{1.5} \times \sqrt{f'_{c}} \]

AASHTO LRFD specifies the modulus of elasticity for concrete in Section 5.4.2.4.

\[ \beta_1 = \text{concrete stress block factor} \]
$T_w = \text{thickness of main reinforcement web}$

$X_{bar} = \text{distance from reinf centroid to edge of reinf toward concrete face}$

The main reinforcement is not symmetric. A term was defined to describe the neutral axis depth of the main reinforcement to the edge of the main reinforcement. See Figure 2 for the illustration.

![Figure 2 – Xbar Illustration](image)

\[ d = \text{depth of reinforcement (M +)} = h - X_{bar} - C_{bottom} \]
\[ d' = \text{depth of reinforcement (M –)} = h - X_{bar} - C_{top} \]
\[ CR_{dia} = \text{cross rod diameter} \]
\[ b_{cr} = \text{cross rod spacing} \]

Holes are placed through the webs of the main bars. The area of the main bars must be corrected for the area of the hole.
Balanced Reinforcement Ratio (AASHTO FRP Equation 2-7) (ACI 440 Equation 8-3) is shown below. Concrete crushing and FRP rupture occur simultaneously the balanced reinforcement ratio.

\[
\rho_{fb} = \text{bal. reinfr. ratio} = 0.85 \beta_1 \frac{f'_c E_{bar} \varepsilon_c}{f_{fd} E_{bar} \varepsilon_c + f_{fd}}
\]

The positive and negative moment strip widths are from Table 4.6.2.1.3-1 (AASHTO LRFD, 2007).

\[
b_{w_{pos}}(in) = \text{positive beam strip width} = 26 + 6.6 \times s(ft)
\]

\[
b_{w_{neg}}(in) = \text{negative beam strip width} = 48 + 3.0 \times s(ft)
\]

<table>
<thead>
<tr>
<th>Design Example: Material Properties (1.5” I bars at 4 inches on center)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f'_c) = 5000 psi</td>
</tr>
<tr>
<td>(A_g) = 0.4021 in²</td>
</tr>
<tr>
<td>(f_{fd\ bar}) = 72800 psi</td>
</tr>
<tr>
<td>(f_{fd\ cr}) = 112280 psi</td>
</tr>
<tr>
<td>(E_{bar}) = 4750 ksi</td>
</tr>
<tr>
<td>(E_{cr}) = 6920 ksi</td>
</tr>
<tr>
<td>(\varepsilon_c) = 0.003</td>
</tr>
<tr>
<td>(\varepsilon_{fd\ bar}) = 0.007</td>
</tr>
</tbody>
</table>
\[ E_c = 33000 \times 0.150 \frac{kcf^{1.5} \times \sqrt{5} ksi}{ksi} = 4.29 \times 10^3 ksi = 4.29 \times 10^6 psi \]

\[ \beta_1 = 0.80 \]

\[ T_w = 0.16 \text{ in} \]

\[ X_{bar} = 0.67 \text{ in} \]

\[ C_{top} = 1.5 \text{ in} \]

\[ C_{bottom} = 1.0 \text{ in} \]

\[ h = 8 \text{ in} \]

\[ d = 8 \text{ in} - 0.67 \text{ in} - 1.0 \text{ in} = 6.33 \text{ in} \]

\[ d' = 8 \text{ in} - 0.67 \text{ in} - 1.5 \text{ in} = 5.83 \text{ in} \]

\[ CR_{dia} = 0.50 \text{ in} \]

\[ b_{cr} = 4.0 \text{ in} \]

\[ A_p = 0.50 \text{ in} \times 0.16 \text{ in} = 0.08 \text{ in}^2 \]

\[ A_n = 0.4021 \text{ in}^2 - 0.08 \text{ in}^2 = 0.3221 \text{ in}^2 \]

\[ A_{cr} = \frac{\pi}{4} (0.50 \text{ in})^2 = 0.1963 \text{ in}^2 \]

\[ \rho_{fb} = 0.85 \times 0.80 \frac{5000 \text{ psi}}{72800 \text{ psi}} \frac{4.75 \times 10^6 \text{ psi} \times 0.003}{4.75 \times 10^6 \text{ psi} \times 0.003 + 72800 \text{ psi}} = 0.0076 \]

\[ b_{w pos} = 26 + 6.6 \times \frac{93.7}{12} \text{ ft} = 77.54 \text{ in} \]

\[ b_{w neg} = 48 + 3 \times \frac{93.7}{12} \text{ ft} = 71.43 \text{ in} \]
5.2 **AASHTO FRP and ACI 440 – Temperature and Shrinkage**

The minimum temperature and shrinkage reinforcement ratio is determined in accordance with AASHTO FRP Section 2.11.5 and ACI 440 Chapter 10. The lower and upper bound limits are 0.0014 and 0.0036.

\[
\rho_{f,ts} = \min temp and shrinkage reinforcement ratio = 0.0018 \frac{60000 \text{psi}}{f_{fd}} \frac{29000 \text{ksi}}{E_f}
\]

The top and bottom layers of Gridform are the same size. The total slab thickness and gross area of main reinforcement are used to check temperature and shrinkage. Both layers of reinforcement are used to calculate the temperature and shrinkage reinforcement ratios.

\[
\rho_{f,ts} = \text{main bar temp and shrink reinf ratio} = \frac{2 \times A_g}{h \times b_s}
\]

\[
\rho_{cr,ts} = \text{cross rod temp and shrink reinf ratio} = \frac{2 \times A_{cr}}{h \times b_{cr}}
\]

**Design Example: Temperature and Shrinkage Reinforcement Design Check**

\[
\rho_{f,ts \ bar \ min} = 0.0018 \frac{60000 \text{ psi}}{72800 \text{ psi}} \frac{29000 \text{ ksi}}{4750 \text{ ksi}} = 0.0091 \text{ max controls} = 0.0036
\]

\[
\rho_{f,ts} = \frac{2 \times 0.4021 \text{ in}^2}{8 \text{ in} \times 4 \text{ in}} = 0.0251
\]

The required reinforcement ratio for the main bars is less than the temperature and shrinkage reinforcement ratio.

**Design Check OK**

\[
\rho_{f,ts \ cr \ min} = 0.0018 \frac{60000 \text{ psi}}{112280 \text{ psi}} \frac{29000 \text{ ksi}}{6920 \text{ ksi}} = 0.0040 \text{ max controls} = 0.0036
\]

\[
\rho_{cr,ts} = \frac{2 \times 0.196 \text{ in}^2}{8 \text{ in} \times 4 \text{ in}} = 0.0123
\]
The required reinforcement ratio for the cross rods is less than the temperature and shrinkage reinforcement ratio.

Design Check OK

5.3 **AASHTO FRP and ACI 440 – Positive and Negative Moment**

AASHTO FRP (Section 2.9.3) and ACI 440 (Section 8.2) contain the flexural design equations. Both codes require a minimum amount of flexural reinforcement to prevent failure of flexural elements after cracking occurs. Only tensile reinforcement is included in the flexural reinforcement ratio and the tensile strength of concrete is ignored. FRP is not permitted as compression reinforcement.

\[
A_{f,\text{min}} \geq \max(4.9 \sqrt{f_{c}^{'},330}) \frac{b \times d}{f_{\text{fa}}} 
\]

\[
\rho_{f} = \text{reinf.\,ratio} = \frac{A_{n}}{b \times d} 
\]

The tensile stress in reinforcement at failure is located in the following equation.

\[
f_{f} = \sqrt{\left(\frac{(E_{\text{bar}}\varepsilon_{c})^2}{4} + \frac{0.85\beta_{1}f_{c}^{'}}{\rho_{f}}E_{\text{bar}}\varepsilon_{c} - \frac{1}{2}E_{\text{bar}}\varepsilon_{c} \right)} \leq f_{\text{fa}} 
\]

The flexural resistance factor is a determined using the reinforcement ratio and balanced reinforcement ratio. FRP fails in a brittle manner. Concrete crushing is a slightly more favorable failure mode than FRP rupture, which is reflected in the strength reduction factor. The strength reduction factor for FRP reinforced concrete slabs controlled by concrete crushing is not the same as concrete slabs reinforced with steel until the actual reinforcement ratio to balanced reinforcement ratio is 1.4 due to the concrete strength. Concrete strength will often be stronger than the specified concrete strength. Consequently, the failure mode could be concrete crushing in design and FRP rupture in the bridge deck due to the concrete strength being higher than the design concrete strength.

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\[ \phi = 0.55 \quad \text{for} \quad \rho_f \leq \rho_{fb} \]

\[ \phi = 0.3 + 0.25 \frac{\rho_f}{\rho_{fb}} \quad \text{for} \quad \rho_{fb} < \rho_f < 1.4\rho_{fb} \]

\[ \phi = 0.65 \quad \text{for} \quad \rho_f \geq 1.4\rho_{fb} \]

The nominal flexural resistance is dependent on the failure mode. Note that the area of the bar is per bar spacing. The nominal strength is calculated per bar spacing and modified to determine the capacity per foot. If concrete crushing controls \((\rho_f > \rho_{fb})\), then the nominal strength is given in the following equation:

\[
M_n = A_n \times f_f \left( d - \frac{1}{2} \left( \frac{A_n \times f_f}{0.85 \times f'_{c} \times b_z} \right) \right)
\]

If FRP rupture controls \((\rho_f < \rho_{fb})\), then the nominal strength is given in the following equation:

\[
M_n = A_n \times f_f \left( d - \frac{1}{2} \left( \beta_1 \times \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{f_d}} \times d \right) \right)
\]

Design Example: Positive Moment Design Check

\[
A_{f,\min} \geq \max \left( 4.9\sqrt{5000 \text{ psi}}, 330 \right) \frac{4 \times 6.33 \text{ in}}{72800 \text{ psi}} = 0.1205 \text{in}^2
\]

The net area is greater than the minimum area.

\[
\rho_f = \frac{0.3221 \text{ in}^2}{4 \text{ in} \times 6.33 \text{ in}} = 0.0127
\]

\[
f_f = \sqrt{\frac{(475000 \text{ psi} \times 0.003)^2}{4} + \frac{0.85 \times 0.8 \times 5000 \text{ psi}}{0.0127} - \frac{1}{2}475000 \text{ psi} \times 0.003}
\]

\[
= \frac{54999 \text{ psi}}{72800 \text{ psi}} \leq 728000 \text{ psi}
\]
\[ \frac{\rho_f}{\rho_{fb}} = \frac{0.0127}{0.0076} = 1.66 \]

Failure Mode: Concrete Crushing

\[ \phi = 0.65 \quad \text{for} \quad \rho_f \geq 1.4\rho_{fb} \]

The nominal moment is calculated per bar spacing and adjusted to a per foot basis.

\[ M_n = 0.3221 \text{in}^2 \times 54999 \text{ psi} \left( 6.33 \text{ in} - \frac{1}{2} \left( \frac{0.3221 \text{in}^2 \times 54999 \text{ psi}}{0.85 \times 500 \text{ psi} \times 4 \text{ in}} \right) \right) \frac{1 \text{ kip}}{1000 \text{ lb}} \frac{1 \text{ ft}}{12 \text{ in}} \frac{12 \text{ in}}{4 \text{ in}} \]

\[ = 25.73 \text{ kip ft ft} \]

\[ \phi M_n = 16.72 \text{ kip ft ft} \geq M_u = 10.59 \text{ kip ft ft} \]

**Design Check OK**

Design Example: Negative Moment Design Check

\[ A_{f,\text{min}} \geq \max \left( 4.9\sqrt{5000 \text{ psi}}, 330 \right) \frac{4 \times 5.83 \text{ in}}{72800 \text{ psi}} = 0.1110 \text{in}^2 \]

The net area is greater than the minimum area.

\[ \rho'_{f} = \frac{0.3221 \text{ in}^2}{4 \text{ in} \times 5.83 \text{ in}} = 0.0138 \]

\[ f_f = \sqrt{\frac{(4750000 \text{ psi} \times 0.003)^2}{4} + \frac{0.85 \times 0.8 \times 5000 \text{ psi}}{0.0138}} \frac{4750000 \text{ psi} \times 0.003}{4750000 \text{ psi} \times 0.003} \]

\[- \frac{1}{2} 4750000 \text{ psi} \times 0.003 = 52529 \text{ psi} \leq 728000 \text{ psi} \]

\[ \frac{\rho'_f}{\rho_{fb}} = \frac{0.0138}{0.0076} = 1.81 \]

Failure Mode: Concrete Crushing
The nominal moment is calculated per bar spacing and adjusted to a per foot basis.

$$M_n = -0.3221 in^2 \times 52529 \text{ psi} \left( 5.83 in - \frac{1}{2} \left( \frac{0.3221 in^2 \times 52529 \text{ psi}}{0.85 \times 500 \text{ psi} \times 4 \text{ in}} \right) \right) \frac{1 \text{ kip}}{1000 \text{ lb}} \frac{1 \text{ ft}}{12 \text{ in}} \frac{12 \text{ in}}{b}$$

$$= -22.56 \text{ kip } \frac{ft}{ft}$$

$$\phi M_n = -14.66 \text{ kip } \frac{ft}{ft} \geq M_u = -9.76 \text{ kip } \frac{ft}{ft}$$

**Design Check OK**

### 5.4 AASHTO FRP and ACI 440 – Punching Shear

Test data indicates that the AASHTO FRP / ACI 440 punching shear equation is conservative. The University of Wisconsin-Madison punching shear equation is recommended for analysis. New terms are introduced below. Gridform cross rods are spaced at 4 inches on center and are at the same depth as the main reinforcement. Only tensile reinforcement (bottom layer) is incorporated into the punching shear reinforcement ratios. The shear strength reduction factor for FRP reinforced concrete is 0.75.

$$\rho_{cr} = \text{cross rod reinf. ratio} = \frac{A_{cr}}{b_{cr} \times d}$$

The modulus of elasticity for the main reinforcement and the cross rods vary. Therefore, the modulus of elasticity is averaged and weighted with the reinforcement ratio in each direction.

$$E_{f, ps} = \frac{E_{bar} \rho_{bar} + E_{cr} \rho_{cr}}{\rho_{bar} + \rho_{cr}}$$

The reinforcement ratio for punching shear is averaged and weighted with the modulus of elasticity in each direction.
\[ \rho_{ps} = \frac{E_{\text{bar}} \rho_{\text{bar}} + E_{\text{cr}} \rho_{\text{cr}}}{E_{\text{bar}} + E_{\text{cr}}} \]

Section 9.2 (ACI 440) presents the modular ratio. The punching shear modulus of elasticity and punching shear reinforcement ratio are used for the neutral axis depth to reinforcement depth for punching shear.

\[ \eta_f = \text{modular ratio} = \frac{E_f}{E_c} \]

\[ k = \text{ratio of n.a. depth to reinf. depth} = \sqrt{2 \times \rho_f \times \eta_f + (\rho_f \times \eta_f)^2} - \rho_f \times \eta_f \]

\[ c = \text{n.a. depth} = k \times d \]

The punching shear capacity is presented in Section 9.4 (ACI 440, 2006).

\[ u_{0.5} = \text{critical offset perimeter at } 0.5d = 2 \times (w + l) + 0.5 \times 8 \times d \]

\[ V_{c,ps} = \text{ACI 440 punching shear capacity} = 10 \sqrt{f'_{c}} \times u_{0.5} \times c \]

The ultimate load for punching shear factored with the largest live load factor and the dynamic load allowance factor.

\[ V_u = LF \times (1 + IM) \times P_{LL} \]

**Design Example: AASHTO FRP and ACI 440 Punching Shear Design Check**

\[ \rho_f = 0.0127 \]

\[ \rho_{cr} = \frac{0.1963 \text{ in}^2}{4 \text{ in} \times 6.33 \text{ in}} = 0.0078 \]

\[ E_{f,ps} = \frac{4570000 \text{ psi} \times 0.0127 + 6920000 \text{ psi} \times 0.0078}{0.0127 + 0.0078} = 5572000 \text{ psi} \]

\[ \rho_{ps} = \frac{4570000 \text{ psi} \times 0.0127 + 6920000 \text{ psi} \times 0.0078}{4570000 \text{ psi} + 6920000 \text{ psi}} = 0.0098 \]
\[ \eta_f = \frac{5572000 \text{ psi}}{4290000 \text{ psi}} = 1.30 \]
\[ k = \sqrt{2 \times 0.0098 \times 1.30 + (0.0098 \times 1.30)^2} - 0.0098 \times 1.30 = 0.147 \]
\[ c = 0.147 \times 6.33 \text{ in} = 0.93 \text{ in} \]
\[ u_{0.5} = 2 \times (10 \text{ in} + 20 \text{ in}) + 0.5 \times 8 \times 6.33 \text{ in} = 85.32 \text{ in} \]
\[ V_{c_{ps}} = 10\sqrt{5000 \text{ psi} \times 85.32 \text{ in} \times 0.93 \text{ in}} = 56.22 \text{ kips} \]
\[ \phi V_{c_{ps}} = 42.16 \text{ kip} \]
\[ V_u = 1.75 \times 1.33 \times 16 \text{ kip} = 37.24 \text{ kip} \]

**Design Check OK**

### 5.5 University of Wisconsin Punching Shear

The University of Wisconsin punching shear equation is based on the research and was calibrated for the Gridform system. See Brunton (2009), Ringelstetter (2006), and Jacobson (2004) for corresponding test data. Refer to Section 5.4 for punching shear reinforcement ratio and strength reduction factor.

\[ u_{1.5} = \text{critical offset perimeter at } 1.5d = 2 \times (w + l) + 1.5 \times 8 \times d \]

\[ V_{n_{uw_{ps}}} = \text{UW punching shear capacity} = 55.33 \frac{3^{\frac{3}{4}} \rho_{ps} \times f'_c}{\sqrt[4]{d}} u_{1.5} \times d \]

**Design Example: University of Wisconsin-Madison Punching Shear Design Check**

\[ \rho_{ps} = 0.0098 \]
\[ u_{1.5} = 2 \times (20 \text{ in} + 10 \text{ in}) + 1.5 \times 8 \times 6.33 \text{ in} = 135.96 \text{ in} \]
\[ V_{n_{uw_{ps}}} = 55.33 \frac{3^{\frac{3}{4}} \times 0.0098 \times 5000 \text{ psi}}{\sqrt[4]{6.33 \text{ in}}} 135.96 \text{ in} \times 6.33 \text{ in} = 109.77 \text{ kips} \]
\[ \phi V_{tuwps} = 82.33 \text{ kips} \]
\[ V_u = 1.75 \times 1.33 \times 16 \text{ kip} = 37.24 \text{ kip} \]

Design Check OK

### 5.6 AASHTO FRP – Positive Moment Region Fatigue and Creep Rupture

AASHTO FRP (Section 2.7.3) limits the stress in the FRP for non-multigirder situations. Fatigue and creep rupture need to be checked against the allowable stress for the Service I load combination. The main reinforcement ratio and modular ratio are used from this point on in the design program. The ratio of the neutral axis depth to reinforcement depth for the main bars is used for fatigue and creep rupture and all following limit states in the program that require the ratio.

\[ \eta_f = \text{modular ratio} = \frac{E_f}{E_c} \]

\[ k = \text{ratio of n.a. depth to reinf. depth} = \sqrt{2 \times \rho_f \times \eta_f + \left( \rho_f \times \eta_f \right)^2} - \rho_f \times \eta_f \]

\[ I_{cr} = \text{cracked moment of inertia} = \frac{b \times d^3}{3} k^3 + \eta_f \times A_n \times d^2 (1 - k)^2 \]

\[ M_s = \text{AASHTO LRFD service I moment} \]

\[ f_{fs} = \text{stress in tensile reinf. at service conditions} = \frac{\eta_f \times d (1 - k)}{I_{cr}} M_s \]

The service stress in reinforcement is calculated assuming that the strain is linearly proportional to the distance from the neutral axis and the tensile strength of concrete is neglected.

\[ 0.2 \times f_{fd \text{ bar}} = \text{limiting stress in tensile reinf.} \]

Design Example: Positive Moment Region Fatigue and Creep Rupture Design Check

\[ \eta_f = \frac{475000 \text{ psi}}{429000 \text{ psi}} = 1.11 \]
The cracked moment of inertia is calculated per foot of width.

\[
l_{cr} = \left( \frac{4.00 \, \text{in} \times (6.33 \, \text{in})^3}{3} - 0.154^3 + 1.11 \times 0.3221 \, \text{in}^2 \times (6.33 \, \text{in})^2 (1 - 0.154)^2 \right) \frac{12 \, \text{in}}{4 \, \text{in}}
\]
\[
= 34.41 \, \frac{\text{in}^4}{\text{ft}}
\]

\[M_s = \text{AASHTO LRFD service I moment} = 6.19 \, \text{kip} \frac{ft}{ft}\]

\[f_{f,s} = \frac{1.11 \times 6.33 \, \text{in} (1 - 0.154)}{34.41 \, \frac{\text{in}^4}{\text{ft}}} \times 6.19 \, \text{kip} \frac{ft}{ft} \times \frac{12 \, \text{in}}{1 \, \text{ft}} = 12.80 \, \text{ksi}\]

\[f_{f,s} \leq 0.2 \times f_{fd\,bar} = 0.2 \times 72.8 \, \text{ksi} = 14.56 \, \text{ksi}\]

**Design Check OK**

### 5.7 AASHTO FRP – Negative Moment Region Fatigue and Creep Rupture

\[
k = \text{ratio of n.a. depth to reinf. depth} = \sqrt{2 \times \rho'_f \times \eta_f + \left( \rho'_f \times \eta_f \right)^2} - \rho'_f \times \eta_f
\]

\[l_{cr} = \text{cracked moment of inertia} = \frac{b \times d't^3}{3} - k^3 + \eta_f \times A_n \times d't^2 (1 - k)^2
\]

\[f_{f,s} = \text{stress in tensile reinf. at service conditions} = \frac{\eta_f \times d (1 - k)}{l_{cr}} M_s
\]

\[0.2 \times f_{fd\,bar} = \text{limiting stress in tensile reinf.}
\]

**Design Example: Negative Moment Region Fatigue and Creep Rupture Design Check**

\[\eta_f = 1.11\]
\[ \rho_f' = 0.0138 \]

\[ k = \sqrt{2 \times 0.0138 \times 1.11 + (0.0138 \times 1.11)^2} - 0.0138 \times 1.11 = 0.160 \]

\[ l_{cr} = \left( \frac{4.00 \text{ in} \times (5.83 \text{ in})^3}{3} \times 0.160^3 + 1.11 \times 0.3221 \text{ in}^2 \times (5.83 \text{ in})^2 (1 - 0.160)^2 \right) \frac{12 \text{ in}}{4 \text{ in}} \]
\[ = 28.92 \text{ in}^4/\text{ft} \]

\[ M_s = \text{AASHTO LRFD service I moment} = -5.75 \text{ kip ft}/\text{ft} \]

\[ f_{f,s} = \frac{1.11 \times 5.83 \text{ in}(1 - 0.160)}{28.92 \text{ in}^4/\text{ft}} \times 5.75 \text{ kip ft}/\text{ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 12.95 \text{ ksi} \]

\[ f_{f,s} \leq 0.2 \times f_{fd, bar} = 0.2 \times 72.8 \text{ ksi} = 14.56 \text{ ksi} \]

**Design Check OK**

### 5.8 AASHTO FRP / ACI 440 Cracking

(AASHTO FRP Section 2.9.3.4) and (ACI 440 Section 8.3)

Traditionally a limiting crack width is required by codes to prevent steel corrosion. FRP does not have corrosion problem. AASHTO FRP and ACI 440 limit the crack width only when aesthetics is a concern. The AASHTO FRP and ACI 440 crack width estimate equations are the same. The equations are a result of testing and are not theoretically derived. The crack width equation has a bond coefficient \((k_b)\), which accounts for the smaller bond/friction between FRP and concrete compared to steel and concrete. A bond coefficient equal to 1 assumes the same bond between FRP and concrete and steel and concrete. The recommended bond coefficient from AASHTO FRP and ACI 440 is 1.4. The recommended bond coefficient is 1.40 for Gridform.
5.8.1 **ACI 440 – Positive Moment Region Cracking**

\[
\beta = \text{ratio} = \frac{h - k \times d}{d - k \times d}
\]

\[
d_c = \text{tension face to reinf. centroid} = h - d
\]

\[
w_s = 2 \frac{f_{f,s}}{E_{\text{bar}}} \beta \times k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}
\]

**Design Example: Positive Moment Region Design Check**

\[
M_s = \text{AASHTO LRFD service I moment} = 6.19 \text{ kip} \frac{ft}{ft}
\]

\[
f_{f,s} = 12.80 \text{ ksi}
\]

\[k = 0.154\]

See fatigue and creep rupture limit state for service stress in FRP and k.

\[
\beta = \frac{8.00 \text{ in} - 0.154 \times 6.33 \text{ in}}{6.33 \text{ in} - 0.154 \times 6.33 \text{ in}} = 1.31
\]

\[
d_c = 8.00 \text{ in} - 6.33 \text{ in} = 1.67 \text{ in}
\]

\[
w_s = 2 \frac{12.80 \text{ ksi}}{4750 \text{ ksi}} 1.31 \times 1.4 \sqrt{\left(1.67 \text{ in}\right)^2 + \left(\frac{4 \text{ in}}{2}\right)^2} = 0.026 \text{ in}
\]

**Design Check No Good**

5.8.2 **ACI 440 – Negative Moment Region Cracking**

\[
\beta = \text{ratio} = \frac{h - k \times d'}{d' - k \times d'}
\]

\[
d_c = \text{tension face to reinf. centroid} = h - d'
\]
Design Example: Negative Moment Region Design Check

\[ M_s = AASHTO\ LRFD\ service\ I\ moment = -5.75\ kip\ \frac{ft}{ft} \]

\[ f_{f,s} = 12.95\ ksi \]

\[ k = 0.160 \]

See fatigue and creep rupture limit state for service stress in FRP and k.

\[ \beta = \frac{8.00\ in - 0.160 \times 5.83\ in}{5.83\ in - 0.160 \times 5.83\ in} = 1.44 \]

\[ d_c = 8.00\ in - 5.83\ in = 2.17\ in \]

\[ w_s = 2 \frac{12.95\ ksi}{4750\ ksi} \times 1.44 \times 1.4 \times \sqrt{(2.17\ in)^2 + \left(\frac{4\ in}{2}\right)^2} = 0.033\ in \]

Design Check No Good

5.9 AASHTO FRP - Concrete Stress Limit States

AASHTO FRP Section 2.9.3.6 limits the stress in the concrete for service load conditions. The maximum allowable stress in the concrete to 45% of the compressive strength for concrete subjected to the Service I load combination (AASHTO LRFD, 2007).

\[ c = k \times d \]

\[ f_{c,s} = \frac{2 \times A_n \times f_{f,s}}{b_s \times c} \]
\[ M_s = \text{AASHTO LRFD service moment} = 6.19 \, \text{kip ft} \]

\[ f_{fs} = 12.80 \, \text{ksi} \]

\[ k = 0.154 \]

See fatigue and creep rupture limit state for service stress in FRP and \( k \).

\[ c = 0.154 \times 6.33\text{in} = 0.98\text{in} \]

\[
f_{cs} = \frac{2 \times 0.3221\text{in}^2 \times 12.80 \, \text{ksi}}{4\text{in} \times 0.98\text{in}} \times \frac{1000\text{lbs}}{1\text{kip}} = 2109\text{psi} \]

\[ f_{allow} = 0.45 \times f'_c = 0.45 \times 5000\text{psi} = 2250\text{psi} \]

**Design Check OK**

Design Example: Negative Moment Region Concrete Stress Design Check

\[ M_s = \text{AASHTO LRFD service moment} = -5.75 \, \text{kip ft} \]

\[ f_{fs} = 12.95 \, \text{ksi} \]

\[ k = 0.160 \]

See fatigue and creep rupture limit state for service stress in FRP and \( k \).

\[ c = 0.160 \times 5.83\text{in} = 0.93\text{in} \]

\[
f_{cs} = \frac{2 \times 0.3221\text{in}^2 \times 12.95 \, \text{ksi}}{4\text{in} \times 0.93\text{in}} \times \frac{1000\text{lbs}}{1\text{kip}} = 2231\text{psi} \]

\[ f_{allow} = 0.45 \times f'_c = 0.45 \times 5000\text{psi} = 2250\text{psi} \]

**Design Check OK**
5.10 AASHTO FRP and ACI 440 – Deflection

Deflection is based on the live point load, the lane loading, and the uniform live load on a fixed–fixed span. The deflection calculation determines the maximum of the live lane load deflection and the live point load deflection, which is added to the uniform live load to determine the total live load deflection. The effective moment of inertia was determined using the Service I moment. The effective stiffness of concrete members reinforced with FRP is the almost the same as concrete members reinforced with steel. The cracked moment of inertia for concrete reinforced with steel was modified with a reduction term ($\beta_d$) to account for the reduced tension stiffening for lightly FRP reinforced concrete members.

\[ l_g = \text{g}r\text{o}ss \text{ of inertia} = \frac{1}{12} b \times h^3 \]
\[ M_{cr} = \text{cracking moment} = \frac{6\sqrt{f'c} \times l_g}{1/2 \ h} \]
\[ \beta_d = \frac{1}{5} \rho_f \leq 1.0 \]
\[ l_e = \text{effective of inertia} = \left( \frac{M_{cr}}{M_a} \right)^3 \beta_d \times l_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] l_{cr} \leq l_g \]
\[ P_{LL} = (1 + IM) \times P_{LL} \]
\[ w_{LL} = b \times LL_{lj} \]
\[ w_{LL\_lane} = b \times LL_{lane} \]
\[ \Delta_{LL} = \max \left( \frac{P_{LL} \times s^3}{192 E_c \times I''_e}, \frac{w_{LL\_lane} \times s^4}{384 E_c \times I''_e} \right) + \frac{w_{LL} \times s^4}{384 E_c \times I''_e} \]

Design Example: Deflection Design Check

\[ l_{cr} = 34.41 \ \text{in}^4 \]

\[ l_t \]
\[ l_g = \frac{1}{12} 12\text{in} \times (8\text{in})^3 = 512 \frac{\text{in}^4}{\text{ft}} \]

\[ M_{cr} = \frac{6\sqrt{5000\text{psi} \times 512 \frac{\text{in}^4}{\text{ft}}}}{1/2 \times 8\text{in}} \frac{1\text{kip}}{1000\text{lbs}} \frac{1\text{ft}}{12\text{in}} = 4.53 \frac{\text{kip}}{\text{ft}} \]

\[ M_a = M_s = 6.19 \frac{\text{kip}}{\text{ft}} \]

\[ \frac{\beta_f}{\rho_{fb}} = 1.66 \]

\[ \beta_d = \frac{1}{5} 1.66 = 0.33 \leq 1.0 \]

\[ l_e = \left( \left( \frac{4.53 \frac{\text{kip}}{\text{ft}}}{6.19 \ \frac{\text{kip}}{\text{ft}}} \right)^3 0.33 \times 512 \frac{\text{in}^4}{\text{ft}} + \left[ 1 - \left( \frac{4.53 \frac{\text{kip}}{\text{ft}}}{6.19 \ \frac{\text{kip}}{\text{ft}}} \right)^3 \right] 34.41 \frac{\text{in}^4}{\text{ft}} \right) \times 77.54\text{in} \times \frac{1\text{ft}}{12\text{in}} \]

\[ = 566\text{in}^4 \leq \left\{ 512 \frac{\text{in}^4}{\text{ft}} \right\} \times 77.54\text{in} \times \frac{1\text{ft}}{12\text{in}} = 3308\text{in}^4 \]

\[ E_c l_e = 4.29 \times 10^6 \text{psi} \times 566\text{in}^4 \times \frac{k\text{ip}}{1000\text{lbs}} = 2.43 \times 10^6 \text{kip} \times \text{in}^2 \]

\[ P_{LL} = (1.33) \times 16\text{kip} = 21.28 \text{kips} \]

\[ w_{LL} = 77.54\text{in} \times 20\text{psf} \times \frac{1 \text{kip}}{1000\text{lbs}} \times \frac{\text{ft}^2}{144\text{in}^2} = 0.0108 \frac{\text{kip}}{\text{in}} \]

\[ w_{LL, lane} = 77.54\text{in} \times 64\text{psf} \times \frac{1 \text{kip}}{1000\text{lbs}} \times \frac{\text{ft}^2}{144\text{in}^2} = 0.0345 \frac{\text{kip}}{\text{in}} \]

\[ \Delta_{LL} = \max \left( \frac{21.28 \text{kips} \times (93.7\text{in})^3}{192(2.43 \times 10^6 \text{kip} - \text{in}^2)} , \frac{0.0345 \frac{\text{kip}}{\text{in}} \times (93.7\text{in})^4}{384(2.43 \times 10^6 \text{kip} - \text{in}^2)} \right) \]

\[ + \frac{0.0108 \frac{\text{kip}}{\text{in}} \times (93.7\text{in})^4}{384(2.43 \times 10^6 \text{kip} - \text{in}^2)} \]
\[ \Delta_{LL} = \max(0.0375 \text{ in}, 0.00284 \text{ in}) + 0.000892 \text{ in} = 0.0375 \text{ in} + 0.000892 \text{ in} \]

\[ = 0.038 \text{ in} \Delta_{LL, limit} = 0.117 \text{ in} \]

**Design Check OK**

6 **INPUT-CALC PRINT SHEET**

This sheet contains all of the information on the Input and Calc sheets in a printable format.

7 **A4-1 SHEET**

This sheet contains Table A4-1 (AASHTO LRFD, 2007). The HL-93 live load moments are located in the table for spans between 4 feet and 15 feet. The span is rounded up to the nearest inch for the span length and the offset distance to the critical negative moment section is rounded down to the nearest inch. The HL-93 live load moments per foot of width may be slightly higher than actually calculated.

8 **LINK SHEET**

This sheet contains the cells that are linked to the program.

**REFERENCES**


American Concrete Institute Committee 440. (2006). "Guide for the Design and Construction of Structural Concrete Reinforced with FRP Bars (440.1R-06)", American Concrete Institute, MI, USA.


